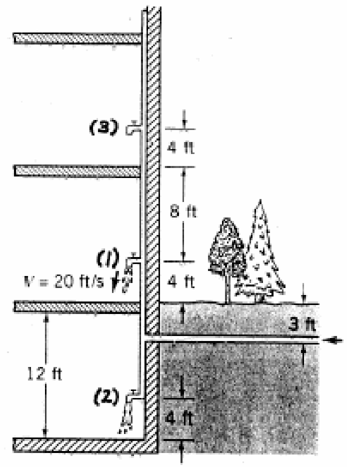
3.14

3.14 Water flows from the faucet on the first floor of the building shown in Fig. P3.14 with a maximum velocity of 20 ft/s. For steady inviscid flow, determine the maximum water velocity from the basement faucet and from the faucet on the second floor (assume each floor is 12 ft tall).



■ FIGURE P3.14

$$\frac{p}{\gamma} + \frac{V^2}{2g} + z = \text{constant}$$

Thus, $\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$ with $p_2 = p_1 = 0$ (free jet) and $V_1 = 20\,ft/s$, $z_1 = 4\,ft$, $z_2 = -8\,ft$

or

$$\frac{(20\,\frac{ft}{s})^2}{2(32.2\,\frac{ft}{s^2})} + 4\,ft = \frac{V_2^2}{2(32.2\,\frac{ft}{s^2})} + (-8\,ft)$$

or $\underline{\underline{V_2 = 34.2\,\frac{ft}{s}}}$

and $\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + z_3$ with $p_3 = p_1 = 0$ (free jet) and $V_1 = 20\,\frac{ft}{s}$, $z_1 = 4\,ft$, $z_3 = 16\,ft$

or

$$\frac{(20\,\frac{ft}{s})^2}{2(32.2\,\frac{ft}{s^2})} + 4\,ft = \frac{V_3^2}{2(32.2\,\frac{ft}{s^2})} + 16\,ft$$

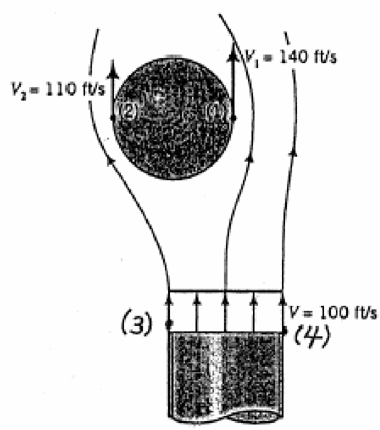
or

$$V_3 = \sqrt{20^2 - 2(32.2)(12)} = \sqrt{-373}$$

Impossible! No flow from second floor faucet.

3.16

3.16 A 100 ft/s jet of air flows past a ball as shown in Video V3.1 and Fig. P3.16. When the ball is not centered in the jet, the air velocity is greater on the side of the ball near the jet center [point (1)] than it is on the other side of the ball [point (2)]. Determine the pressure difference, $p_2 - p_1$, across the ball if $V_1 = 140$ ft/s and $V_2 = 110$ ft/s. Neglect gravity and viscous effects.



■ FIGURE P3.16

The Bernoulli equation from point (3) to (2) and (4) to (1) with gravity neglected gives

$$p_3 + \tfrac{1}{2}\rho V_3^2 = p_2 + \tfrac{1}{2}\rho V_2^2 \quad \text{and} \quad p_4 + \tfrac{1}{2}\rho V_4^2 = p_1 + \tfrac{1}{2}\rho V_1^2$$

But $p_3 = p_4 = 0$ and $V_3 = V_4$

Thus, even though points (1) and (2) are not on the same streamline,

$$p_1 + \tfrac{1}{2}\rho V_1^2 = p_2 + \tfrac{1}{2}\rho V_2^2$$

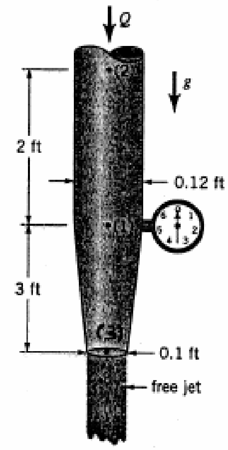
or

$$p_1 - p_2 = \tfrac{1}{2}\rho(V_1^2 - V_2^2) = \tfrac{1}{2}\left(0.00238\,\frac{\text{slugs}}{\text{ft}^3}\right)\left[\left(140\,\frac{\text{ft}}{\text{s}}\right)^2 - \left(110\,\frac{\text{ft}}{\text{s}}\right)^2\right]$$

$$= 8.93\,\frac{\text{slugs}}{\text{ft}\cdot\text{s}^2} = \underline{\underline{8.93\,\frac{\text{lb}}{\text{ft}^2}}}$$

3.25

3.25 Water flows steadily downward through the pipe shown in Fig. P3.25. Viscous effects are negligible, and the pressure gage indicates the pressure is zero at point (1). Determine the flowrate and the pressure at point (2).



■ FIGURE P3.25

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_3}{\gamma} + z_3 + \frac{V_3^2}{2g}$$

where $z_1 = 3\text{ ft}$, $z_3 = 0$, $p_1 = p_3 = 0$

and

$$V_1 = \frac{A_3}{A_1} V_3 = \left(\frac{\frac{\pi}{4}(0.1\text{ ft})^2}{\frac{\pi}{4}(0.12\text{ ft})^2}\right) V_3 = 0.694\, V_3$$

Thus,

$$\frac{(0.694)^2 V_3^2}{2(32.2\text{ ft/s}^2)} + 3\text{ ft} = \frac{V_3^2}{2(32.2\text{ ft/s}^2)} \quad \text{or} \quad V_3 = 19.3\,\frac{\text{ft}}{\text{s}}$$

so that

$$Q_3 = A_3 V_3 = \frac{\pi}{4}(0.1\text{ ft})^2\left(19.3\,\frac{\text{ft}}{\text{s}}\right) = \underline{\underline{0.152\,\frac{\text{ft}^3}{\text{s}}}}$$

Also,

$$\frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g} = \frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g}$$

where $p_1 = 0$ and since $A_1 = A_2$ it follows that $V_2 = V_1$

Thus,

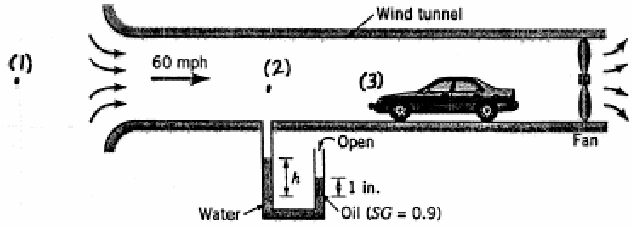
$$z_2 - z_1 = -\frac{p_2}{\gamma} \quad \text{or} \quad \frac{p_2}{\gamma} = -2\text{ ft}$$

or

$$p_2 = -2\text{ ft}\left(62.4\,\frac{\text{lb}}{\text{ft}^3}\right) = \underline{\underline{-125\,\frac{\text{lb}}{\text{ft}^2}}}$$

3.27

3.27 Air is drawn into a wind tunnel used for testing automobiles as shown in Fig. P3.27. (a) Determine the manometer reading, h, when the velocity in the test section is 60 mph. Note that there is a 1-in. column of oil on the water in the manometer. (b) Determine the difference between the stagnation pressure on the front of the automobile and the pressure in the test section.



■ FIGURE P3.27

(a) $\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$

where

$z_1 = z_2$, $p_1 = 0$, and $V_1 \approx 0$

Thus, with $V_2 = 60\ mph = 88\ \frac{ft}{s}$,

$\frac{p_2}{\gamma} = -\frac{V_2^2}{2g}$ or

$p_2 = -\frac{1}{2}\rho V_2^2 = -\frac{1}{2}(0.00238\ \frac{slugs}{ft^3})(88\ \frac{ft}{s})^2 = -9.22\ \frac{lb}{ft^2}$

But $p_2 + \gamma_{H_2O}h - \gamma_{oil}(\frac{1}{12}ft) = 0$ where $\gamma_{oil} = 0.9\gamma_{H_2O} = 0.9(62.4\ \frac{lb}{ft^3})$
$= 56.2\ \frac{lb}{ft^3}$

Thus,

$-9.22\frac{lb}{ft^2} + 62.4\frac{lb}{ft^3}(h\ ft) - 56.2\frac{lb}{ft^3}(\frac{1}{12}ft) = 0$, or $\underline{\underline{h = 0.223\ ft}}$

(b) $\frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g} = \frac{p_3}{\gamma} + z_3 + \frac{V_3^2}{2g}$

where

$z_2 = z_3$ and $V_3 = 0$

Thus,

$\frac{p_2}{\gamma} + \frac{V_2^2}{2g} = \frac{p_3}{\gamma}$ or

$p_3 - p_2 = \frac{1}{2}\rho V_2^2 = \frac{1}{2}(0.00238\ \frac{slugs}{ft^3})(88\ \frac{ft}{s})^2 = \underline{\underline{9.22\ \frac{lb}{ft^2}}}$

3.30

3.30 Water flows through the pipe contraction shown in Fig. P3.30. For the given 0.2-m difference in manometer level, determine the flowrate as a function of the diameter of the small pipe, D.

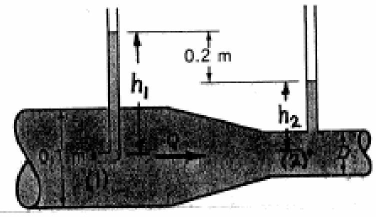


FIGURE P3.30

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{or with } z_1 = z_2 \text{ and } V_1 = 0$$

$$V_2 = \sqrt{2g\,\frac{(p_1 - p_2)}{\gamma}}$$

but $p_1 = \gamma h_1$ and $p_2 = \gamma h_2$ so that $p_1 - p_2 = \gamma(h_1 - h_2) = 0.2\gamma$

Thus,

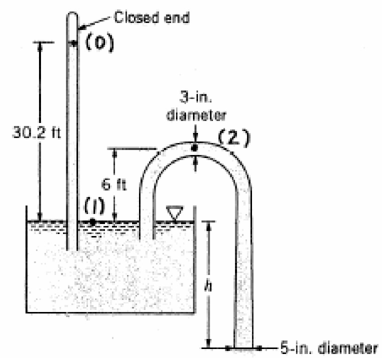
$$V_2 = \sqrt{2g\,\frac{0.2\gamma}{\gamma}} = \sqrt{2g(0.2)}$$

or

$$Q = A_2 V_2 = \frac{\pi}{4}D^2 V_2 = \frac{\pi}{4}D^2\sqrt{2(9.81)(0.2)} = \underline{\underline{1.56\,D^2\ \frac{m^3}{s}}} \text{ when } D \sim m$$

3.39

3.39 Water is siphoned from the tank shown in Fig. P3.39. The water barometer indicates a reading of 30.2 ft. Determine the maximum value of h allowed without cavitation occurring. Note that the pressure of the vapor in the closed end of the barometer equals the vapor pressure.



■ FIGURE P3.39

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{where } p_1 = 0,\ V_1 = 0,\ p_2 = p_{vapor}$$

Thus, $\quad z_1 = 0,\ z_2 = 6\ ft$

$$0 = \frac{p_{vapor}}{\gamma} + \frac{V_2^2}{2g} + 6\ ft$$

but $p_0 + 30.2\ ft\ \gamma = p_1$ or since $p_0 = p_{vapor}$, $\frac{p_{vapor}}{\gamma} = -30.2\ ft$

Hence,

$$0 = -30.2\ ft + \frac{V_2^2}{2g} + 6\ ft \quad \text{or} \quad \frac{V_2^2}{2g} = 24.2\ ft \quad \text{or} \quad V_2^2 = \left[2\left(32.2\frac{ft}{s^2}\right)(24.2\ ft)\right]$$

Thus,

$$V_2 = 39.5\ \frac{ft}{s}$$

Since $V_3 A_3 = V_2 A_2$, $\quad V_3 = \frac{A_2}{A_3} V_2 = \frac{D_2^2}{D_3^2} V_2 = \left(\frac{3\ in.}{5\ in.}\right)^2 \left(39.5\ \frac{ft}{s}\right)$

or

$$V_3 = 14.2\ \frac{ft}{s}$$

However,

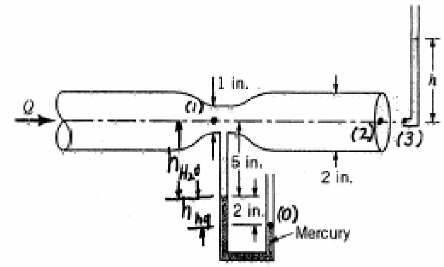
$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + z_3 \quad \text{or} \quad V_3 = \sqrt{2gh}$$

Thus,

$$14.2\ \frac{ft}{s} = \sqrt{2\left(32.2\frac{ft}{s^2}\right) h\ ft} \quad \text{or} \quad \underline{\underline{h = 3.13\ ft}}$$

3.45

3.45 Water flows through a converging–diverging nozzle as shown in Fig. P3.45. Determine (**a**) the volumetric flowrate, Q, through the nozzle and (**b**) the height, h, of the water in the Pitot tube inserted into the free jet. Viscous effects are negligible.



■ FIGURE P3.45

(a) From the Bernoulli equation,

$$p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2, \text{ where } z_1 = z_2 \text{ and } p_2 = 0 \qquad (1)$$

Also, $p_1 = p_0 - \gamma_{Hg} h_{Hg} - \gamma_{H_2O} h_{H_2O}$, where $p_0 = 0$ so that

$$p_1 = -13.6(62.4\,\text{lb/ft}^3)(\tfrac{2}{12}\,\text{ft}) - 62.4\,\text{lb/ft}^3(\tfrac{5}{12}\,\text{ft}) = -167\,\text{lb/ft}^2$$

In addition, $A_1 V_1 = A_2 V_2$ or $V_1 = \left(\frac{D_2}{D_1}\right)^2 V_2 = \left(\frac{2\,\text{in.}}{1\,\text{in.}}\right)^2 V_2 = 4V_2$

Hence, Eq. (1) becomes

$$-167\,\text{lb/ft}^2 + \tfrac{1}{2}(1.94\,\text{slugs/ft}^3)(4V_2)^2 = \tfrac{1}{2}(1.94\,\text{slugs/ft}^3)V_2^2$$

or

$$V_2 = 3.39\,\text{ft/s}$$

Thus, $Q = A_2 V_2 = \frac{\pi}{4}\left(\frac{2}{12}\,\text{ft}\right)^2(3.39\,\text{ft/s}) = \underline{\underline{0.0739\,\text{ft}^3/\text{s}}}$

(b) From the Bernoulli equation,

$$p_2 + \tfrac{1}{2}\rho V_2^2 + \gamma z_2 = p_3 + \tfrac{1}{2}\rho V_3^2 + \gamma z_3, \text{ where } p_2 = 0,\ z_2 = 0,\ V_3 = 0, \text{ and } z_3 = 0$$

Thus,

$$\tfrac{1}{2}\rho V_2^2 = p_3, \text{ where } p_3 = \gamma h \text{ so that}$$

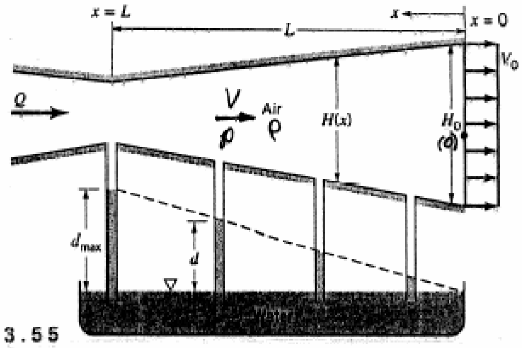
$$\tfrac{1}{2}(1.94\,\text{slugs/ft}^3)(3.39\,\text{ft/s})^2 = (62.4\,\text{lb/ft}^3)h$$

or

$$h = \underline{\underline{0.179\,\text{ft} = 2.14\,\text{in.}}}$$

3.55

3.55 Air flows steadily through a converging–diverging rectangular channel of constant width as shown in Fig. P3.55 and Video V3.6. The height of the channel at the exit and the exit velocity are H_0 and V_0, respectively. The channel is to be shaped so that the distance, d, that water is drawn up into tubes attached to static pressure taps along the channel wall is linear with distance along the channel. That is, $d = (d_{max}/L)\,x$, where L is the channel length and d_{max} is the maximum water depth (at the minimum channel height; $x = L$). Determine the height, $H(x)$, as a function of x and the other important parameters.



■ FIGURE P3.55

$$p + z\gamma + \frac{1}{2}\rho V^2 = p_0 + z_0\gamma + \frac{1}{2}\rho V_0^2 \quad \text{where } \rho = \text{air density}$$

where

$$z = z_0, \quad p_0 = 0, \quad p = -\gamma_{H_2O}\, d = -\gamma_{H_2O}\frac{d_{max}}{L}x$$

Thus,

$$-\gamma_{H_2O}\frac{d_{max}}{L}x + \frac{1}{2}\rho V^2 = \frac{1}{2}\rho V_0^2$$

But

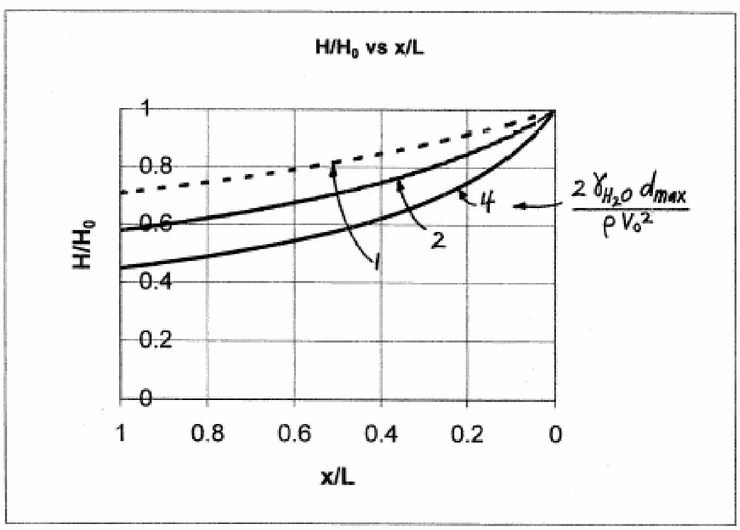
$$AV = A_0 V_0, \quad \text{or} \quad V = \frac{A_0}{A}V_0 = \frac{H_0}{H}V_0 \quad \text{so that}$$

$$-\gamma_{H_2O}\frac{d_{max}}{L}x + \frac{1}{2}\rho\left(\frac{H_0}{H}V_0\right)^2 = \frac{1}{2}\rho V_0^2$$

or

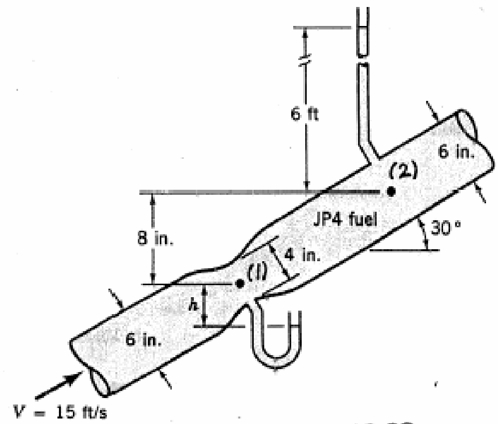
$$\frac{H}{H_0} = \frac{1}{\sqrt{1 + \left(\frac{2\gamma_{H_2O}\, d_{max}}{\rho V_0^2}\right)\frac{x}{L}}}$$

Typical shapes are shown below.



3.68

3.68 JP-4 fuel ($SG = 0.77$) flows through the Venturi meter shown in Fig. P3.68 with a velocity of 15 ft/s in the 6-in. pipe. If viscous effects are negligible, determine the elevation, h, of the fuel in the open tube connected to the throat of the Venturi meter.



■ FIGURE P3.68

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{where } z_1 = 0,\ z_2 = \frac{8}{12}\text{ ft}, \quad (1)$$

and $V_2 = 15\ \text{ft/s}$

Also, $A_1 V_1 = A_2 V_2$

or

$$V_1 = \frac{A_2}{A_1} V_2 = \left(\frac{D_2}{D_1}\right)^2 V_2 = \left(\frac{6\text{ in.}}{4\text{ in.}}\right)^2 \left(15\ \frac{\text{ft}}{\text{s}}\right) = 33.75\ \frac{\text{ft}}{\text{s}}$$

Thus, with $\frac{p_2}{\gamma} = 6\text{ ft}$ Eq. (1) becomes

$$\frac{p_1}{\gamma} + \frac{(33.75\ \frac{\text{ft}}{\text{s}})^2}{2(32.2\ \frac{\text{ft}}{\text{s}^2})} = 6\text{ ft} + \frac{(15\ \frac{\text{ft}}{\text{s}})^2}{2(32.2\ \frac{\text{ft}}{\text{s}^2})} + \frac{8}{12}\text{ ft}$$

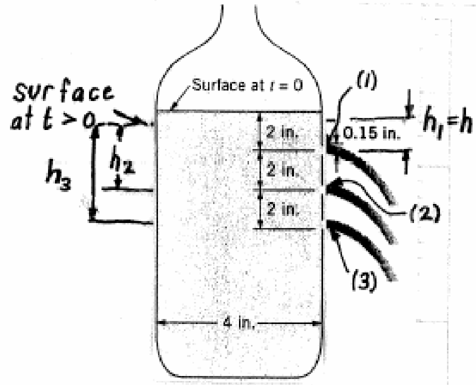
or

$$\frac{p_1}{\gamma} = -7.53\text{ ft}$$

But $\frac{p_1}{\gamma} = -h$ so that $h = \underline{\underline{7.53\text{ ft}}}$

3.20

3.20 Pop (with the same properties as water) flows from a 4-in. diameter pop container that contains three holes as shown in Fig. P3.20 (see Video 3.5). The diameter of each fluid stream is 0.15 in., and the distance between holes is 2 in. If viscous effects are negligible and quasi-steady conditions are assumed, determine the time at which the pop stops draining from the top hole. Assume the pop surface is 2 in. above the top hole when $t = 0$. Compare your results with the time you measure from the video.



■ FIGURE P3.20

$$Q = Q_1 + Q_2 + Q_3 = -A_T \frac{dh}{dt}$$

where $Q_i = V_i A_i = \sqrt{2gh_i}\, A_i$ $(i=1,2,3)$ and $A_1 = A_2 = A_3 = \frac{\pi}{4}\left(\frac{0.15}{12}\text{ ft}\right)^2 = 1.227 \times 10^{-4}\text{ ft}^2$

$$A_T = \frac{\pi}{4}\left(\frac{4}{12}\text{ ft}\right)^2 = 0.0873\text{ ft}^2$$

Thus,

$$\sqrt{2g}\, A_1\left[\sqrt{h_1} + \sqrt{h_2} + \sqrt{h_3}\right] = -A_T \frac{dh}{dt},$$ where $h_1 = h$, $h_2 = h + L$, $h_3 = h + 2L$ and $L = 2\text{ in.}$

Hence,

$$-\left(\sqrt{2g}\, A_1 / A_T\right)\int_0^t dt = \int_L^0 \frac{dh}{\left(\sqrt{h} + \sqrt{h+L} + \sqrt{h+2L}\right)}$$

where t is the time it take for the free surface to reach the upper hole $(h=0)$.

or

$$t = \frac{A_T}{A_1\sqrt{2g}}\int_0^L \frac{dh}{\left(\sqrt{h} + \sqrt{h+L} + \sqrt{h+2L}\right)}$$

$$= \frac{0.0873\text{ ft}^2}{\left(1.227 \times 10^{-4}\text{ ft}^2\right)\left[(2)(32.2\text{ ft/s}^2)\right]^{1/2}}\int_0^L \frac{dh}{\left(\sqrt{h} + \sqrt{h+L} + \sqrt{h+2L}\right)}$$

Thus,

$$t = 88.7\int_0^L \frac{dh}{\left(\sqrt{h} + \sqrt{h+L} + \sqrt{h+2L}\right)}$$ where $L = \frac{2}{12}\text{ ft} = 0.1667\text{ ft}$

Note: With L in feet, this equation gives t in seconds.

(con't)

3.20 (con't)

The numerical value of the integral is obtained by using the trapezoidal rule since the closed form analytical solution is not given in integral tables. The EXCEL spread sheet used for this is given below.

$$t = 88.7\int_0^L f(h)\,dh \quad \text{where } f(h) = \frac{1}{(\sqrt{h} + \sqrt{h+L} + \sqrt{h+2L})}$$

$$\approx 88.7\left[\frac{1}{2}\sum_{i=1}^{20}(f_i + f_{i+1})(h_{i+1} - h_i)\right] = \left(88.7\ \frac{s}{\sqrt{ft}}\right)\left[0.120\sqrt{ft}\right] = \underline{\underline{10.7\,s}}$$

h, in.	h, ft	f(h), 1/ft$^{1/2}$	(1/2)*(f_i + f_{i+1})*(h_{i+1} - h_i), ft$^{1/2}$	i
0.0	0.0000	1.015	0.00804	1
0.1	0.0083	0.914	0.00743	2
0.2	0.0167	0.870	0.00711	3
0.3	0.0250	0.837	0.00686	4
0.4	0.0333	0.810	0.00665	5
0.5	0.0417	0.786	0.00646	6
0.6	0.0500	0.764	0.00629	7
0.7	0.0583	0.745	0.00614	8
0.8	0.0667	0.728	0.00600	9
0.9	0.0750	0.712	0.00587	10
1.0	0.0833	0.697	0.00575	11
1.1	0.0917	0.684	0.00564	12
1.2	0.1000	0.671	0.00554	13
1.3	0.1083	0.659	0.00544	14
1.4	0.1167	0.647	0.00535	15
1.5	0.1250	0.637	0.00526	16
1.6	0.1333	0.627	0.00518	17
1.7	0.1417	0.617	0.00510	18
1.8	0.1500	0.608	0.00503	19
1.9	0.1583	0.599	0.00496	20
2.0	0.1667	0.591		21
	Sum of column = integral =		0.12011	

Thus, t = 88.7*0.12011 = 10.7 s

3.31

3.31 Water flows through the pipe contraction shown in Fig. P3.31. For the given 0.2-m difference in the manometer level, determine the flowrate as a function of the diameter of the small pipe, D.

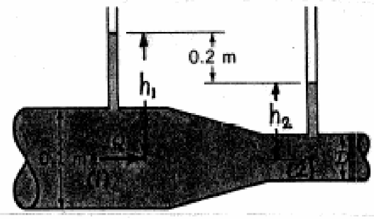


FIGURE P3.31

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{with} \quad A_1 V_1 = A_2 V_2$$

Thus, with $z_1 = z_2$ or $V_2 = \frac{\left(\frac{\pi}{4} D_1^2\right)}{\left(\frac{\pi}{4} D_2^2\right)} V_1 = \left(\frac{0.1}{D}\right)^2 V_1$

$$\frac{p_1 - p_2}{\gamma} = \frac{V_2^2 - V_1^2}{2g} = \frac{\left[\left(\frac{0.1}{D}\right)^4 - 1\right] V_1^2}{2g}$$

but

$p_1 = \gamma h_1$ and $p_2 = \gamma h_2$ so that $p_1 - p_2 = \gamma (h_1 - h_2) = 0.2\gamma$

Thus,

$$\frac{0.2\gamma}{\gamma} = \frac{\left[\left(\frac{0.1}{D}\right)^4 - 1\right] V_1^2}{2g} \quad \text{or} \quad V_1 = \sqrt{\frac{0.2\,(2g)}{\left[\left(\frac{0.1}{D}\right)^4 - 1\right]}}$$

and

$$Q = A_1 V_1 = \frac{\pi}{4}(0.1)^2 \sqrt{\frac{0.2\,(2\,(9.81))}{\left[\left(\frac{0.1}{D}\right)^4 - 1\right]}}$$

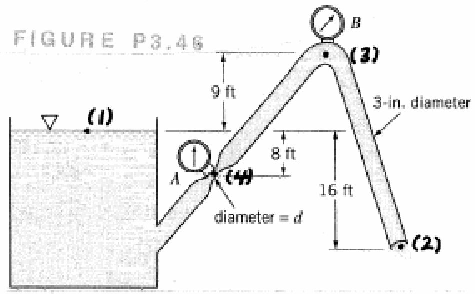
or

$$\underline{\underline{Q = \frac{0.0156\, D^2}{\sqrt{(0.1)^4 - D^4}}}} \ \frac{m^3}{s} \quad \text{when } D \sim m$$

3.46

3.46 Water flows steadily from a large open tank and discharges into the atmosphere through a 3-in.-diameter pipe as shown in Fig. P3.46. Determine the diameter, d, in the narrowed section of the pipe at A if the pressure gages at A and B indicate the same pressure.

FIGURE P3.46



$$p_4 + \frac{1}{2}\rho V_4^2 + \gamma z_4 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2 \text{ , where } z_2 = 0 \text{ and } p_2 = 0$$

Thus, since $p_3 = p_4$

$$p_3 + \frac{1}{2}\rho V_4^2 + \gamma z_4 = \frac{1}{2}\rho V_2^2 \qquad (1)$$

However, $p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2$, where $p_1 = p_2 = V_1 = z_2 = 0$

so that

$$\frac{1}{2}\rho V_2^2 = \gamma z_1 \text{ , or } V_2 = \sqrt{2\frac{\gamma}{\rho} z_1} = \sqrt{2 g z_1} = \left[2(32.2\tfrac{ft}{s^2})(16\,ft)\right]^{\frac{1}{2}} = 32.1\,ft/s$$

But

$$p_3 + \frac{1}{2}\rho V_3^2 + \gamma z_3 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2 \quad \text{where } V_2 = V_3 \text{ since } A_2 = A_3$$

Thus,

$$p_3 = -\gamma z_3 = -(16+9)ft(62.4\,lb/ft^3) = -1560\,lb/ft^2 \qquad (2)$$

From Eqs. (1) and (2):

$$-1560\,\frac{lb}{ft^2} + \frac{1}{2}(1.94\,\frac{slugs}{ft^3})V_4^2 = \frac{1}{2}(1.94\,\frac{slugs}{ft^3})(32.1\,\frac{ft}{s})^2$$

or

$$V_4 = 46.1\,ft/s$$

Since $A_4 V_4 = A_2 V_2$ it follows that

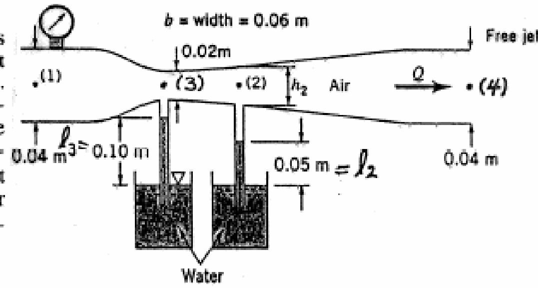
$$\frac{\pi}{4} d^2 V_4 = \frac{\pi}{4} D_2^2 V_2$$

or

$$d = D_2\sqrt{\frac{V_2}{V_4}} = (3\,in.)\sqrt{\frac{32.1\,ft/s}{46.1\,ft/s}} = \underline{\underline{2.50\,in.}}$$

3.51

3.51 Air flows through a Venturi channel of rectangular cross section as shown in **Video V3.6** and Fig. P3.51. The constant width of the channel is 0.06 m and the height at the exit is 0.04 m. Compressibility and viscous effects are negligible. **(a)** Determine the flowrate when water is drawn up 0.10 m in a small tube attached to the static pressure tap at the throat where the channel height is 0.02 m. **(b)** Determine the channel height, h_2, at section (2) where, for the same flowrate as in part **(a)**, the water is drawn up 0.05 m. **(c)** Determine the pressure needed at section (1) to produce this flow.



■ FIGURE P3.51

(a) For steady, inviscid, incompressible flow: ($\gamma = 12.0\ \frac{N}{m^3}$)

(1) $\frac{p_3}{\gamma} + \frac{V_3^2}{2g} = \frac{p_4}{\gamma} + \frac{V_4^2}{2g}$ where $p_4 = 0$, $p_3 = -\gamma_{H_2O}\ell_3 = 9.80\times10^3\frac{N}{m^3}(0.1m)$
$= -980\ \frac{N}{m^2}$

Also, $A_3V_3 = A_4V_4$ so that $V_3 = \frac{(0.04m\times0.06m)}{(0.02m\times0.06m)}V_4 = 2V_4$

Thus, Eqn. (1) becomes

$$\frac{-980\ \frac{N}{m^2}}{12.0\ \frac{N}{m^3}} + \frac{4V_4^2}{2(9.81\frac{m}{s^2})} = \frac{V_4^2}{2(9.81\frac{m}{s^2})} \quad \text{or} \quad V_4 = 23.1\ \frac{m}{s}$$

Hence,

$$Q = A_4V_4 = (0.04m\times0.06m)(23.1\ \tfrac{m}{s}) = \underline{\underline{0.0554\ \tfrac{m^3}{s}}}$$

(2) (b) $\frac{p_2}{\gamma} + \frac{V_2^2}{2g} = \frac{p_4}{\gamma} + \frac{V_4^2}{2g}$ where $p_4 = 0$, $p_2 = -\gamma_{H_2O}\ell_2 = 9.80\times10^3\frac{N}{m^3}(0.05m)$
$= -490\ \frac{N}{m^2}$

From part (a), $V_4 = 23.1\ \frac{m}{s}$

Thus, Eqn. (2) becomes

$$\frac{-490\ \frac{N}{m^2}}{12.0\ \frac{N}{m^3}} + \frac{V_2^2}{2(9.81\frac{m}{s^2})} = \frac{(23.1\ \frac{m}{s})^2}{2(9.81\frac{m}{s^2})} \quad \text{or} \quad V_2 = 36.5\ \frac{m}{s}$$

But $V_2A_2 = V_4A_4$ so that

$$(36.5\tfrac{m}{s})(0.06m)h_2 = (23.1\tfrac{m}{s})(0.06m)(0.04m) \quad \text{or} \quad h_2 = \underline{\underline{0.0253\ m}}$$

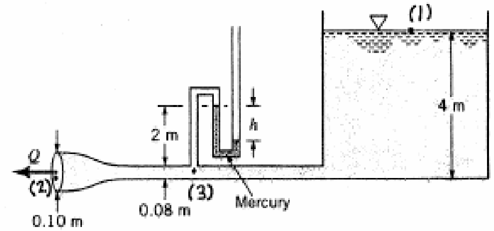
(3) (c) Also, $\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_4}{\gamma} + \frac{V_4^2}{2g}$ where $p_4 = 0$ and $A_1V_1 = A_4V_4$

But since $A_1 = (0.04m\times0.06m) = A_4$ then $V_1 = V_4$ and Eqn. (3) gives

$p_1 = p_4 = \underline{\underline{0}}$

3.53

3.53 Water flows steadily from the large open tank shown in Fig. P3.53. If viscous effects are negligible, determine (a) the flowrate, Q, and (b) the manometer reading, h.



■ FIGURE P3.53

(a) From the Bernoulli equation,

$$p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2, \text{ where } p_1 = p_2 = 0,\ V_1 = 0,\ z_1 = 4m, \text{ and } z_2 = 0.$$

Thus,

$$\gamma z_1 = \frac{1}{2}\rho V_2^2, \text{ or } \rho g z_1 = \frac{1}{2}\rho V_2^2 \text{ so that } V_2 = \sqrt{2 g z_1}$$

or

$$V_2 = \sqrt{2(9.81\,m/s^2)(4m)} = 8.86\,m/s$$

Hence,

$$Q = A_2 V_2 = \frac{\pi}{4}(0.10m)^2(8.86m/s) = \underline{\underline{0.0696\,m^3/s}}$$

(b) From the Bernoulli equation,

$$p_3 + \frac{1}{2}\rho V_3^2 + \gamma z_3 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_3, \text{ where } z_2 = z_3 \text{ and } p_2 = 0$$

so that

$$p_3 = \frac{1}{2}\rho(V_2^2 - V_3^2)$$

Also, $A_2 V_2 = A_3 V_3$ so that $V_3 = \frac{A_2}{A_3} V_2 = \left(\frac{D_2}{D_3}\right)^2 V_2 = \left(\frac{0.1m}{0.08m}\right)^2 8.86 m/s = 13.84 m/s$

Hence,

$$p_3 = \frac{1}{2}(999\,kg/m^3)\left[(8.86m/s)^2 - (13.84m/s)^2\right] = -56{,}500\,N/m^2 \qquad (1)$$

Also, from the manometer,

$$p_3 = -\gamma_{Hg} h + \gamma_{H_2O}(2m + (0.08/2)m)$$
$$= -(133 \times 10^3 N/m^3)h + (9.80 \times 10^3 N/m^3)(2.04m)$$
$$= -133 \times 10^3 h + 1.99 \times 10^4 \ N/m^2, \text{ where } h \sim m \qquad (2)$$

Thus, from Eqs. (1) and (2):

$$-5.65 \times 10^4 N/m^2 = -133 \times 10^3 h + 1.99 \times 10^4 N/m^2$$

or

$$h = \underline{\underline{0.574\,m}}$$

3.60

3.60 Water flows from a large tank as shown in Fig. P3.60. Atmospheric pressure is 14.5 psia and the vapor pressure is 1.60 psia. If viscous effects are neglected, at what height, h, will cavitation begin? To avoid cavitation, should the value of D_1 be increased or decreased? To avoid cavitation, should the value of D_2 be increased or decreased? Explain.

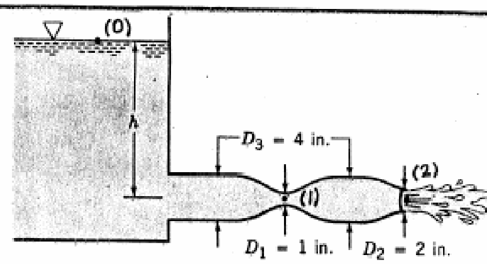


FIGURE P3. 60

$$\frac{p_0}{\gamma} + \frac{V_0^2}{2g} + z_0 = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1$$ where $p_0 = 14.5\ psia$, $p_1 = 1.60\ psia$, $z_0 = h$, $z_1 = 0$, and $V_0 = 0$

Thus,

$$h = \frac{p_1 - p_0}{\gamma} + \frac{V_1^2}{2g} \qquad (1)$$

However,

$$A_1 V_1 = A_2 V_2 \quad \text{or} \quad V_1 = \left(\frac{D_2}{D_1}\right)^2 V_2$$

where

$$\frac{p_0}{\gamma} + \frac{V_0^2}{2g} + z_0 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$ with $p_0 = p_2$ and $z_2 = 0$

Thus,

$$\frac{V_2^2}{2g} = h$$

so that

$$\frac{V_1^2}{2g} = \frac{\left(\frac{D_2}{D_1}\right)^4 V_2^2}{2g} = \left(\frac{D_2}{D_1}\right)^4 h \qquad (2)$$

Combine Eqs. (1) and (2) to obtain

$$h = \frac{p_1 - p_0}{\gamma} + \left(\frac{D_2}{D_1}\right)^4 h$$

or

$$h = \frac{p_0 - p_1}{\gamma\left[\left(\frac{D_2}{D_1}\right)^4 - 1\right]} = \frac{(14.5 - 1.60)\frac{lb}{in.^2}\left(144\frac{in.^2}{ft^2}\right)}{62.4\frac{lb}{ft^3}\left[\left(\frac{2\ in.}{1\ in.}\right)^4 - 1\right]} = \underline{\underline{1.98\ ft}} \qquad (3)$$

From Eq.(3) it is seen that h increases in increasing D_1 and decreasing D_2. Thus, to avoid cavitation (i.e. to have h small enough) $\underline{\underline{D_1 \text{ should be increased and } D_2 \text{ decreased.}}}$

3.66

3.66 Determine the manometer reading, h, for the flow shown in Fig. P3.66.

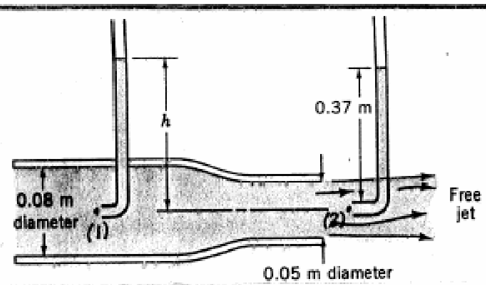


FIGURE P3.66

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$ where $z_1 = z_2$, $V_1 = 0$, and $V_2 = 0$

Thus,

$$p_1 = p_2$$

However, $p_1 = \gamma h$ and $p_2 = \gamma (0.37 m)$ so that

$$h = \underline{\underline{0.37\ m}}$$

3.75

3.75 What diameter orifice hole, d, is needed if under ideal conditions the flowrate through the orifice meter of Fig. P3.75 is to be 30 gal/min of seawater with $p_1 - p_2 = 2.37$ lb/in.^{2}? The contraction coefficient is assumed to be 0.63.

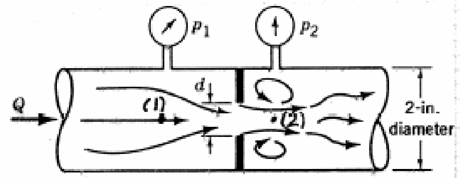


FIGURE P3.75

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \qquad (1)$$

where $z_1 = z_2$, $C_c = 0.63$, and $p_1 - p_2 = 2.37\,psi$

With

$$Q = \left(30\,\frac{gal}{min}\right)\left(\frac{1\,min}{60\,s}\right)\left(\frac{231\,in.^3}{1\,gal}\right)\left(\frac{1\,ft^3}{1728\,in^3}\right) = 0.0668\,\frac{ft^3}{s} \quad \text{and } \gamma = 64.0\,\frac{lb}{ft^3}$$

it follows that

$$V_1 = \frac{Q}{A_1} = \frac{0.0668\,\frac{ft^3}{s}}{\frac{\pi}{4}\left(\frac{2}{12}\,ft\right)^2} = 3.06\,\frac{ft}{s}$$

Thus, Eq(1) gives

$$V_2 = \sqrt{V_1^2 + 2g\left(\frac{p_1 - p_2}{\gamma}\right)} = \sqrt{\left(3.06\,\frac{ft}{s}\right)^2 + 2\left(32.2\,\frac{ft}{s^2}\right)\left(\frac{2.37 \times 144\,\frac{lb}{ft^2}}{64.0\,\frac{lb}{ft^3}}\right)}$$

or

$$V_2 = 18.8\,\frac{ft}{s}$$

Thus, since

$$Q = A_2 V_2 = C_c \frac{\pi}{4} d^2 V_2 \quad \text{it follows that}$$

$$d = \left[\frac{4Q}{\pi C_c V_2}\right]^{1/2} = \left[\frac{4 \times 0.0668\,\frac{ft^3}{s}}{\pi\,(0.63)(18.8\,\frac{ft}{s})}\right]^{1/2} = 0.0847\,ft = \underline{\underline{1.016\ in.}}$$